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Simple unitarity relations among charged current coupling constants

Dan-Di Wu[1]

HEP, Prairie View A&M University, Prairie View, TX 77446-0932, USA

I suggest two new unitarity tests for the quark charged current coupling constants which are $(\sin\gamma/\sin\alpha) = \zeta$*, and* $\frac{\sin 2\gamma}{\sin 2\alpha} = -\frac{(1-\zeta\cos\beta)}{\cos\beta-\zeta}$ *where* ζ *is a CKM parameter defined in the text. These unitarity identities do not suffer some of the multi-value ambiguities. Related problems are discussed. The sign of* Δm_{B_d} *should satisfy the condition* $\Delta m \cos 2\beta < 0$.

The accelerators and detectors for the measurement of CP violation in the B- meson system are close to completion at both SLAC and KEK. Preparation for data acquisition and analysis is intensified[1]. The question of focus is still how to efficiently use limited resources (e.g. the limited data that will be available) to test the standard model (SM), in particular, the unitarity of the charged current coupling constants, which in the SM are grouped into the Cabibbo-Kobayashi-Maskawa (CKM) matrix[2]. In this note I would present two unitarity relations for discussion.

The first new unitarity relation I suggest is

$$\sin\gamma/\sin\alpha = \zeta \tag{1}$$

where α, γ are the unitarity angles discussed widely in the literature and ζ is a CKM parameter that will be defined the next moment. I will also suggest another unitarity raltion in which only $\sin 2\alpha$ and $\sin 2\gamma$ appear in the left-hand side of the equation.

In the standard model, the CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2}$$

[1] E-mail: wu@hp75.pvamu.edu or danwu@flash.net

is believed to be the only coupling constant matrix that couple quarks of different flavors to positively charged W-bosons. It therefore plays an exclusive role in the standard model CP violation phenomenology, since there is not any other charged boson in the standard model, neither a vector nor a scalar meson. This matrix is supposed to be unitary, because of its gauge interaction origin. Beyond the standard model there may be new contributions from additional couplings in some new physics to the processes which exist in the SM, including flavor changing neutral current processes. These new couplings cause an apparent violation of unitarity of the measured effective coupling constants. Unitarity test is therefore important to discovering of possible physics beyond the standard model.

A clever way of testing unitarity is to work on a unitarity triangle, which is defined by one of the orthogonality conditions. A triangle tightly related to B_d mixing and decay is defined by the following unitarity condition

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \tag{3}$$

Regarding the three terms in this equation as three sides, the three outer-angels are defined as

$$\alpha = \arg(B/C), \beta = \arg(C/A), \gamma = \arg(A/B), \tag{4}$$

where A, B, C are defined as

$$A = V_{cd}V_{cb}^*, B = V_{ud}V_{ub}^*, C = V_{td}V_{tb}^*. \tag{5}$$

By definition,

$$\alpha + \beta + \gamma = 2\pi \tag{6}$$

no matter $(A + B + C)$ vanishes or not. However, some bizarre new physics may cause the measured angles dissatisfy Eq (6), (e.g. if the measured angles break the biting-tail relations in (4)) although in terms of unitarity test, Eq (6) is not very useful. What is special of Eq (1) is that it will not stand unless the measured effective parameters are those derived from a unitary CKM matrix.

The relation Eq(1) can be easily proved using the matrix convention of Chen and Wu[3], which reads

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(e^{-i\beta} - \zeta) \\ -\lambda + A^2\zeta\lambda^5 e^{-i\beta} & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 e^{-i\beta} \\ A\zeta\lambda^3 & -A\lambda^2 e^{i\beta} & 1 \end{pmatrix}, \tag{7}$$

where λ and A are the parameters a la Wolfenstein[4]. The currently best fit values for the two parameters are respectively $\lambda = 0.220 \pm 0.002$ and $A = 0.85 \pm 0.07$. β is exactly the β-angle discussed above. ζ in Eq (1) is also defined here. By definition, $\lambda, A, \zeta > 0$ is required. This expression of the CKM matrix is accurate to

the third oreder of λ (i.e. λ^3)), which is good enough for most practical usages.[2]

The three sides of the master triangle discussed above are defined respectively, after a scaling

$$A = -e^{i\beta}, \qquad B = e^{i\beta} - \zeta, \qquad C = \zeta. \tag{8}$$

Combining (8) with (4), we have

$$\sin\alpha = \frac{\sin\beta}{\sqrt{1+\zeta^2-2\zeta\cos\beta}}, \qquad \sin\gamma = \frac{\zeta\sin\beta}{\sqrt{1+\zeta^2-2\zeta\cos\beta}}. \tag{9}$$

Eq (1) therefore follows.

We emphasize that Eq (1) comes from the unitarity of the CKM matrix expressed by $A + B + C = 0$. It does not predict the value of any parameters, unless two of the three parameters in the equation are measured. As mentioned before, Eq (1) relates two physical parameters to a third measurable, which is a CKM parameter. (We will come back to the question of how to measure ζ later on.) However, there are some ambiguities in practice, because what seem likely to be measured in relevant CP violating processes are $\sin2\alpha, \sin2\gamma$ etc. To calculate $\sin\alpha$ from measured $\sin2\alpha$, one obtains two pairs of solutions, each pair involve two solutions with opposite signs. It is therefore useful to provide also another unitarity relation

$$\frac{\sin2\gamma}{\sin2\alpha} = -\frac{(1-\zeta\cos\beta)}{\cos\beta-\zeta} \tag{10}$$

The value of $\cos\beta$ in the right-hand side of Eq (10) will be uniquely defined, if both $\sin2\beta$ and $\sin\beta$ are directly measured from CP violation effects. It is well known that $\sin2\beta$ is exactly the asymmetry in the $B_d \to J/\psi K_S$ decay which was first examined by Bigi, Carter and Sanda [5],

$$a_{B_d\to J/\psi K_S} = sin2\beta. \tag{11}$$

This quantity will be measured cool for both CP violation study and the CKM parametrization. Its signal at the two B-factories will be very clean. The value of $\sin\beta$ can be obtained by the measurement of like-charged lepton asymmetry[6], once ζ is measured

$$a_{ll} = 2\mathrm{Im}(\Gamma_{12}/M_{12})_{B_d}, \tag{12}$$

where a_{ll} is defined as

$$a_{ll} = \frac{N(B^0\bar{B}^0 \to l^+l^+) - N(B^0\bar{B}^0 \to l^-l^-)}{N(B^0\bar{B}^0 \to l^+l^+) + N(B^0\bar{B}^0 \to l^-l^-)}.$$

[2] The expression accurate to the fifth order of λ is

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(e^{-i\beta}-\zeta+\frac{1}{2}\lambda^2e^{-i\beta}) \\ -\lambda+A^2\lambda^5(\zeta e^{-i\beta}-\frac{1}{2}) & 1-\frac{1}{2}\lambda^2-(\frac{1}{8}+\frac{1}{2}A^2)\lambda^4 & A\lambda^2(e^{-i\beta}+\zeta\lambda^2+\frac{1}{2}\lambda^2e^{-i\beta}) \\ A\zeta\lambda^3(1+\frac{1}{2}\lambda^2) & -A\lambda^2e^{i\beta}(1+\frac{1}{2}\lambda^2) & 1-A^2\lambda^4 \end{pmatrix}$$

The right-hand side of Eq (12) is

$$2\mathrm{Im}(\Gamma_{12}/M_{12})_{B_d} = \frac{8\pi}{F_{box}(m_t^2/m_W^2)} \frac{m_c^2}{m_t^2} \frac{\sin\beta}{\zeta}, \tag{13}$$

where $F_{box}(x)$ is a box diagram function defined by Inami and Lim[7].

There is a draw back in Eq (10) concerning the accuracy of the denominator in its right-hand side. If, say, cosβ is measured of value 0.70±0.07 with 10% of accuracy, and ζ is measured of value 0.50±0.10 with 20% of accuracy, then the denominator of the right-hand side of this formula will be 0.20 ±0.12 which is 60% uncertain. Such error enlargement may become worse when the calculations involved are more complicated. We will have a very bad luck in using this formula for unitarity test, if a situation similar to or worse than this happens. There are many other methods to measure sin2β and sinβ, see the BaBar physics book[1].

It might be a good place to clarify the relation between a_{ll} and ϵ_{B_d}. The question is

$$\mathrm{Re}\epsilon_{B_d} ? = \frac{1}{4} a_{ll}.$$

We put a question mark here to make an alarm. To show its invalidity, one finds, from the eigenequations for $\begin{pmatrix} (1+\epsilon) \\ (1-\epsilon) \end{pmatrix}$,

$$\frac{(1+\epsilon)^2}{(1-\epsilon)^2} = \frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}.$$

Defining $M_{12} = -|M_{12}|e^{i\theta}$ (θ is the phase of the mixing mass), $\sigma = \frac{1}{4}\mathrm{Im}(\Gamma_{12}/M_{12})$ (σ also equals to $\frac{1}{2}\langle B_+|B_-\rangle$ and is therefore called the overlap of the two CP eigenstates) and $R = -\mathrm{Re}(\Gamma_{12}/M_{12})$, and noting that σ and R are very small for the neutral B meson systems, one finds,

$$\epsilon_{B_d} = \frac{(1+\sigma) - e^{i\theta}(1-\sigma)}{(1+\sigma) + e^{i\theta}(1-\sigma)}. \tag{14}$$

The value of $\theta = \arg(V_{tb}V_{td}^*)^2$ is CKM convention dependent. For $1 - \cos^2\theta >> \sigma \sim 10^{-3} - 10^{-4}$, one obtains[8]

$$\mathrm{Re}\epsilon_{B_d} = \frac{2\sigma}{1 + \cos\theta + \sigma^2(1-\cos\theta)}.$$

The θ and ϵ_{B_d} values are listed in the following table for different CKM parametrizations.

CKMparametrization	θ	$\mathrm{Re}\epsilon_{B_d}$
Chen – Wu	0	σ
Wolfenstein	$+\arctan(\frac{\eta}{1-\rho})$	$\frac{2\sigma}{(1+\cos\theta)+\sigma^2(1-\cos\theta)}$
Kobayashi – Maskawa	δ	$\frac{2\sigma}{1+\cos\delta+\sigma^2(1-\cos\delta)}$

In the Wolfenstein parametrization, sin2α and sin2β are respectively[1]:

$$\sin 2\alpha = \frac{2\bar{\eta}[\bar{\eta}^2 + \bar{\rho}(\bar{\rho}-1)]}{[\bar{\eta}^2 + (\bar{\rho}-1)^2][\bar{\eta}^2 + \bar{\rho}^2]}, \qquad \sin 2\beta = \frac{-2\bar{\eta}(\bar{\rho}-1)]}{\bar{\eta}^2 + (\bar{\rho}-1)^2}, \tag{15}$$

where $\bar{\eta} = \eta(1-\lambda^2/2)$, $\bar{\rho} = \rho(1-\lambda^2/2)$. Obviousely, it is very difficult to collect information about unitarity from the comparison of these two equations and related measurements.

The value of ζ can be acquired from the mixing mass of the $B_d - \bar{B}_d$ system

$$|\Delta m_{B_d}| = \frac{G_F^2}{6\pi^2}\eta(Bf_B^2)m_t^2 F_{box}(m_t^2/m_W^2)A^2\lambda^6\zeta^2. \tag{16}$$

where η is a QCD correction factor, B is the vacuum insertion, or box constant, and f_B is the B_u to $\mu\bar{\nu}$ decay constant. From this formula the current likelihood of ζ is 0.73 ± 0.29, which is very much dependent on the accuracy of the calculation of $\eta(Bf_B^2)$. cosβ can then be calculated from the $(B \to u + x)$ to $(B \to c + x)$ ratio

$$\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{1+\zeta^2 - 2\zeta\cos\beta}. \tag{17}$$

From this formula, cosβ is obtained, $\cos\beta = 0.9 \pm 0.4$. The uncertainty of cosβ here is mainly from the uncertainty of ζ itself. The right-hand side of Eq (16) can also be written as $\sqrt{(\zeta - \cos\beta)^2 + \sin^2\beta}$, from which one finds that

$$|\sin\beta)| \leq 0.36 \pm 0.07.$$

There is not any error enlargement mechanism involved in Eq (16) because of the smallness of the (b to u)/(b to c) ratio in the left-hand side of this equation, which strongly suggests that cosβ must be positive.

It becomes apparent with the Chen-Wu matrix that the two eigenstates

$$B_+ = \sqrt{\frac{1}{2}}\left[(1+\epsilon)|B_d\rangle + (1-\epsilon)|\bar{B}_d\rangle\right], \qquad B_- = \sqrt{\frac{1}{2}}\left[(1+\epsilon)|B_d\rangle - (1-\epsilon)|\bar{B}_d\rangle\right] \tag{18}$$

are almost pure CP eigenstates, because ϵ here is extremely small. This should be true even if one uses other CKM matrix conventions in which ϵ is large, because CP violation in mixing, a_{ll} is very small anyway. An interesting question one may ask is what the sign of the mass difference Δm of the two eigenstates is, where Δm is defined as

$$\Delta m = m_+ - m_-. \tag{19}$$

The idea was that possible large CP violation in the decay amplitudes might cause the CP even B_d eigenstate to mainly decay into CP odd final states and the CP odd eigenstate into CP even final states. Consequently, the width of B_+ became less than that of B_-. From

$$\Delta m \Delta\gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}), \tag{20}$$

one would then find that $\Delta m > 0$, where $\Delta\gamma$ was the corresponding width difference. Indeed, the leading CKM term in Γ_{12} was also $(V_{tb}V_{td}^*)^2$ as calculated by Hagelin[9] , so the right-hand side of Eq (20) was negative. One knows that Δm_K is negative, but now Δm_{B_d} could be positive[10].

With the Chen-Wu matrix, the discussion is simple, because of the smallness of ϵ_{B_d} in this convention. The criteria[11] for a judgement is

$$\Delta m \mathrm{Re}\Delta_{us}^2 < 0, \tag{21}$$

where

$$\Delta_{us} = V_{cb}V_{td}V_{cd}^*V_{tb}^*, \tag{22}$$

is a relevant quartet rephasing invariant of the CKM matrix. This can be further simplified as

$$\Delta m_{B_d} \cos 2\beta < 0. \tag{23}$$

That means the "anomalous" sign for Δm_{B_d} will appear, if β is in between $\pi/4$ and $3\pi/4$.

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